Eur. Phys. J. A **31**, 854–857 (2007)
DOI 10.1140/epja/i2006-10280-8

THE EUROPEAN PHYSICAL JOURNAL A

Special Article – QNP 2006

Freeze-out of the expanding system

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Received: 18 December 2006
Published online: 20 March 2007 –

Abstract. The freeze-out (FO) of the expanding systems, created in relativistic heavy-ion collisions, is discussed. We start with kinetic FO model, which realizes complete physical FO in a layer of given thickness, and then combine our gradual FO equations with Bjorken-type system expansion into a unified model. We shall see that the basic FO features, pointed out in the earlier works, are not smeared out by the expansion.

PACS. 51.10.+y Kinetic and transport theory of gases – 24.10.Nz Hydrodynamic models – 25.75.-q Relativistic heavy-ion collisions

At the highest energies available nowadays in relativistic heavy-ion collisions at RHIC the total number of the produced particles exceeds 6000, therefore one can expect that the produced system behaves as a "matter" and generates collective effects. Indeed strong collective flow patterns have been measured at RHIC, which suggests that the hydrodynamical models are well justified during the intermediate stages of the reaction: from the time when local equilibrium is reached until the freeze-out (FO), when the hydrodynamical description breaks down. During this FO stage, the matter becomes so dilute and cold that particles stop interacting and stream towards the detectors freely, their momentum distribution freezes out. The FO stage is essentially the last part of a collision process and the main source for observables.

Nowadays, FO is usually simulated in two extreme ways: A) FO on a hypersurface with zero thickness, B) FO described by a volume emission model or hadron cascade, which in principle requires an infinite time and space for a complete FO. At first glance it seems that one can avoid troubles with FO modeling using a hydro+cascade two-module model [1], since in hadron cascades gradual FO is realized automatically. However, in a such a scenario there is an uncertain point, actually uncertain hypersurface, where one switches from hydrodynamical to kinetic modeling. First of all it is not clear how to determine such a hypersurface. This hypersurface in general may have both time-like and space-like parts. Mathematically this problem is very similar to hydro-to-FO phase transition on the infinitely narrow FO hypersurface, therefore for example all the problems discussed for FO on the hypersurface with space-like normal vectors will take place here. Another complication is that while for the post FO domain we have a mixture of non-interacting ideal gases, now for the hadron cascade we should generate distributions for the interacting hadronic gas of all possible species, as a starting point for the further cascade evolution. The volume emission models are based on the kinetic equations [2, 3] defining the evolution of the distribution functions, and therefore these also require to generate initial distribution functions for the interacting hadronic species on some hypersurface.

In this work we present a FO model which allows us to study FO in a layer of any thichness, L, from 0 to ∞, and which connects the pre-FO hydrodynamical quantities, like energy density, e, baryon density, n, with the post-FO distribution function in a relatively simple way. Many building blocks of the model are Lorentz invariant and can be applied to both time-like and space-like FO layers. In this work we are going to include a Bjorken-like expansion in our FO model, in contrast to the older versions [3–7]. In this latter case the FO layer is a domain restricted by two hypersurfaces $\tau = \tau_1$ and $\tau = \tau_1 + L$ (τ is the proper time).

We are going to review briefly all the steps done to derive our model. We will skip all the detailed derivations, refering to the corresponding publication, and will show and discuss only a small part of results, due to the limited space.

Starting from the Boltzmann Transport Equation, introducing two components of the distribution function, f, the interacting, f^i, and the frozen-out, f^f, ones, ($f = f^i + f^f$), and assuming that FO is a directed process (*i.e.* neglecting the gradients of the distribution functions in

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the directions perpendicular to the FO direction with respect to that in the FO direction) we can obtain the following system of equations [6,8]:

$$\frac{\mathrm{d}f^i}{\mathrm{d}s} = -\frac{P_{esc}}{\tau_{FO}} f^i + \frac{f_{eq}(s) - f^i}{\tau_{th}}, \qquad \frac{\mathrm{d}f^f}{\mathrm{d}s} = \frac{P_{esc}}{\tau_{FO}} f^i . \tag{1}$$

The FO direction is defined by the unit vector $\mathrm{d}\sigma_\mu$. FO happens in a layer of given thickness L with two parallel boundary hypersurfaces perpendicular to $\mathrm{d}\sigma_\mu$, and $s = \mathrm{d}\sigma_\mu x^\mu$ is a variable in the FO direction. We work in the reference frame of the front, where $\mathrm{d}\sigma_\mu$ is either $(1,0,0,0)$ for the time-like FO, or $(0,1,0,0)$ for the space-like FO. The τ_{FO} is some characteristic length scale, like mean free path or mean collision time for time-like FO. The rethermalization of the interacting component is taken into account via the relaxation time approximation, where f_i approaches the equilibrated Jüttner distribution, $f_{eq}(s)$, with a relaxation length, τ_{th}. The system (1) can be solved semianalytically in the fast rethermalization limit [6].

According to the above references, the basis of the model, *i.e.* the invariant escape probability within the FO layer of thickness L, for both time-like and space-like normal vectors is given as [6,7,9]

$$P_{esc} = \left(\frac{L}{L - x^\mu \mathrm{d}\sigma_\mu}\right)\left(\frac{p^\mu \mathrm{d}\sigma_\mu}{p^\mu u_\mu}\right)\Theta(p^\mu \mathrm{d}\sigma_\mu), \tag{2}$$

where p^μ is the particle four-momentum, u^μ is the flow velocity. In fact the model based on the escape rate (2) is a generalization of simple kinetic models studied in refs. [3–5], which can be restored in the $L \to \infty$ limit. Here we will concentrate on the time-like case only, where the above Θ-function is unity.

Simple semianalytically solvable FO models studied in [3–7] are missing an important ingredient —the expansion of the freezing-out system. The open question is whether the features of the FO, found in those papers, will survive if the system expasion is included. In this work we present a model which includes both gradual FO and Bjorken-like expansion of the system. And we will see that the answer is "yes" —the basic features of the post FO distrubitions will not be smeared out by the expansion.

First, let us remind the reader the basics of the famous Bjorken model. Bjorken model is one-dimensional in the same sense as discussed before eq. (1) —only the proper time, $\tau = \sqrt{t^2 - x^2}$, gradients are considered. Here the reference frame of the front, $\mathrm{d}\sigma^\mu = (1,0,0,0)$, is the same as the local rest frame, $u^\mu = (1,0,0,0)$. The evolution of the energy density and baryon density is given by the following equations:

$$\frac{\mathrm{d}e}{\mathrm{d}\tau} = -\frac{e+P}{\tau}, \qquad \frac{\mathrm{d}n}{\mathrm{d}\tau} = -\frac{n}{\tau}, \tag{3}$$

where P is the pressure. The initial conditions are given at some $\tau = \tau_0$.

Applying our FO model to such a system, we obtain

$$\mathrm{d}f^i(\tau') = -\frac{\mathrm{d}\tau'}{\tau_{FO}}\frac{L}{L-\tau'} f^i(\tau') + \frac{\mathrm{d}\tau'}{\tau_{th}}\left[f_{eq}(\tau') - f^i(\tau')\right], \tag{4}$$

$$\mathrm{d}f^f(\tau') = +\frac{\mathrm{d}\tau'}{\tau_{FO}}\frac{L}{L-\tau'} f^i(\tau'), \tag{5}$$

where FO begins at $\tau = \tau_1$ and $\tau' = \tau - \tau_1$. Taking the fast rethermalization limit, similarly to what is done in [5], we can obtain simplified equations for f^i, which is a thermal distribution $f^i(\tau) = f_{eq}(\tau)$, f^f as well as for e^i, n^i and e^f, n^f:

$$\frac{\mathrm{d}e^i}{\mathrm{d}\tau'} = -\frac{e^i}{\tau_{FO}}\frac{L}{L-\tau'}, \qquad \frac{\mathrm{d}n^i}{\mathrm{d}\tau'} = -\frac{n^i}{\tau_{FO}}\frac{L}{L-\tau'}, \tag{6}$$

$$\frac{\mathrm{d}e^f}{\mathrm{d}\tau'} = +\frac{e^i}{\tau_{FO}}\frac{L}{L-\tau'}, \qquad \frac{\mathrm{d}n^f}{\mathrm{d}\tau'} = +\frac{n^i}{\tau_{FO}}\frac{L}{L-\tau'}. \tag{7}$$

Now the idea is to create a system of equations which could describe a fireball which simultaneously expands and freezes out. Let us put our two components ($e = e^i + e^f$) into the first equation of (3) and do some simple algebra:

$$\frac{\mathrm{d}e^i}{\mathrm{d}\tau} + \frac{\mathrm{d}e^f}{\mathrm{d}\tau} = -\frac{e^i + P^i}{\tau} - \frac{e^f}{\tau} - \frac{e^i}{\tau_{FO}}\frac{L}{L-\tau'} + \frac{e^i}{\tau_{FO}}\frac{L}{L-\tau'}, \tag{8}$$

where last two terms add up to zero; the free component, of course, has no pressure. So far our eq. (8) is completely identical to the first equation of (3). Our assumption is that our system evolves in such a way that eq. (8) is satisfied as a system of two separate equations for interacting and free components [10]:

$$\frac{\mathrm{d}e^i}{\mathrm{d}\tau} = -\frac{e^i + P^i}{\tau} - \frac{e^i}{\tau_{FO}}\frac{L}{L+\tau_1-\tau}, \tag{9}$$

$$\frac{\mathrm{d}e^f}{\mathrm{d}\tau} = -\frac{e^f}{\tau} + \frac{e^i}{\tau_{FO}}\frac{L}{L+\tau_1-\tau}. \tag{10}$$

Similarly we can obtain equations for baryon density [10]:

$$\frac{\mathrm{d}n^i}{\mathrm{d}\tau} = -\frac{n^i}{\tau} - \frac{n^i}{\tau_{FO}}\frac{L}{L+\tau_1-\tau}, \tag{11}$$

$$\frac{\mathrm{d}n^f}{\mathrm{d}\tau} = -\frac{n^f}{\tau} + \frac{n^i}{\tau_{FO}}\frac{L}{L+\tau_1-\tau}. \tag{12}$$

Thus, finally, we have the following simple model of fireball created in relativistic heavy-ion collision.

Initial state, $\tau = \tau_0$: $e(\tau_0) = e_0$, $n(\tau_0) = n_0$.

Phase I, pure Bjorken hydrodynamics, $\tau_0 \le \tau \le \tau_1$,

$$e(\tau) = e_0\left(\frac{\tau_0}{\tau}\right)^{1+c_o^2}, \quad n(\tau) = n_0\left(\frac{\tau_0}{\tau}\right), \tag{13}$$

where $P = c_o^2 e$ —equation of state (EoS) in general form.

Phase II, Bjorken expansion and gradual FO, $\tau_1 \le \tau \le \tau_1 + L$,

$$e^i(\tau) = e_0\left(\frac{\tau_0}{\tau}\right)^{1+c_o^2}\left(\frac{L+\tau_1-\tau}{L}\right)^{L/\tau_{FO}}, \tag{14}$$

$$n^i(\tau) = n_0\left(\frac{\tau_0}{\tau}\right)\left(\frac{L+\tau_1-\tau}{L}\right)^{L/\tau_{FO}}. \tag{15}$$

With these last equations we have completely determined the evolution of the interacting component [10]. Knowing

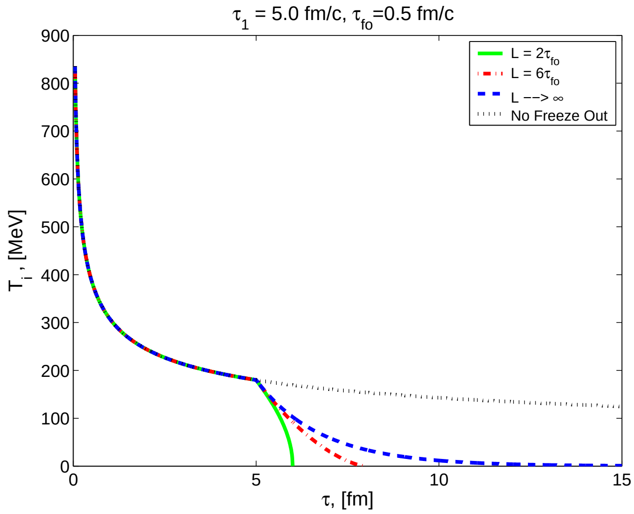

Fig. 1. Evolution of the temperature of the interacting matter for different FO layers. $T_i(\tau_0 = 0.05\,\mathrm{fm}) = 835\,\mathrm{MeV}$, $T_{FO} = 180\,\mathrm{MeV}$. "No Freeze-Out" means that we used standard Bjorken hydrodynamics even in phase II.

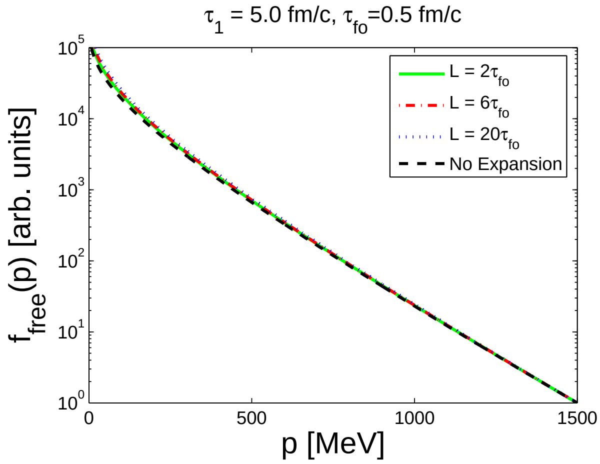

Fig. 2. Final post-FO distribution for different FO layers as a function of the momentum in the FO direction, $p = p^x$ in our case ($p^y = p^z = 0$). The initial conditions are specified in the text. The "No Expansion" curve is given by the analytical expression [5]: $f^f(p) = -\frac{4}{(2\pi)^3} Ei(-\frac{p}{T_{FO}})$.

$e^i(\tau)$ and EoS, we can find the temperature, $T_i(\tau)$. Due to the symmetry of the system $u_i^\mu(\tau) = u^\mu(\tau_0) = (1,0,0,0)$. Finally, $f^i(\tau)$ is a thermal distribution with given $T_i(\tau)$, $n^i(\tau)$, $u_i^\mu(\tau)$.

However, the most interesting for us is the free component, which is the source of the observables. Equations (9), (10) give us the evolution of the e_f and n_f, and one can easily check that these two equations are equivalent to the following equation for the distribution function:

$$\frac{\mathrm{d}f^f}{\mathrm{d}\tau} = -\frac{f^f}{\tau} + \frac{f^i}{\tau_{FO}} \frac{L}{L+\tau_1-\tau}\,. \qquad (16)$$

The measured post-FO spectra are given by $f^f(L+\tau_1)$.

Aiming for a qualitative illustration of the FO process, we show below the results for the massless ideal gas without conserved charges with Jüttner equilibrated distribution ($P^i = e^i/3$, $e^i = \frac{3}{\pi^2}T_i^4$, $f^i(\tau,|\boldsymbol{p}|) = \frac{1}{(2\pi)^3}e^{-|\boldsymbol{p}|/T_i(\tau)}$). We have taken the following values of the parameters: $\tau_0 = 0.05\,\mathrm{fm}$, $T_i(\tau_0) = 835\,\mathrm{MeV}$; $\tau_1 = 5\,\mathrm{fm}$, $T_i(\tau_1) = T_{FO} = 180\,\mathrm{MeV}$; $\tau_{FO} = 0.5\,\mathrm{fm}$ and we present results for different values of FO time L.

Figure 1 shows the evolution of the temperature of the interacting matter. As was already shown in [5,7] the final post-FO particle distributions are non-equilibrated distributions, which deviate from thermal ones particularly in the low-momentum region. By introducing and varying the thickness of the FO layer, L, we are strongly affecting the evolution of the interacting component, see fig. 1, but we again see the universality of the final post-FO distribution: for $L > 2\tau_{FO}$ it already looks very close to that for an infinitely long FO calculations: see fig. 2. The inclusion of the expansion into our consideration does not smear out this very important feature of FO. More results can be found in [10].

In our opinion these results may justify the use of FO hypersurface in hydrodynamical models for heavy-ion collisions, but with a proper non-thermal post FO distributions. If the FO layer is thick enough, say $L > 2\tau_{FO}$, then it does not matter how thick the FO layer was; we do not need to model the FO dynamics in details. Once we have a good parameterization of the post-FO spectrum (still asymmetric, non-thermal), for example the analytical post-FO distribution obtained in ref. [5] (see the caption to fig. 2), then the parameters of this distribution can be found from the conservation laws, as is usually done for sharp FO, with some volume scaling factor to effectively account for the expansion during FO. It is important to always check the non-decreasing entropy condition [10,11] to see whether such a process is physically possible.

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